Modeling and Analysis of Qualitative Systems Based on a New Fuzzy Inference Approach[1]

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Abstract

Qualitative modeling of technical processes may be accomplished by dynamic fuzzy systems. A new inference method with interpolating rules is proposed as an essential basis for the analysis of this class of systems. Using this approach, the system output is dependent on both an interpolating rule derived from the fuzzy input and the fuzzy input itself. A simple example shows the typical behavior of such dynamic fuzzy systems and leads to a stability definition.

Keywords: dynamic fuzzy systems, inference with interpolating rules, stability analysis of dynamic fuzzy systems

1 Introduction

Most control engineering applications of fuzzy logic are based on a set of rules with fuzzy premises and fuzzy conclusions. To describe complex processes qualitatively, a fuzzy output dependent on fuzzy input variables has to be calculated. There exist various reasoning methods [1,4] with particular interpretation of the fuzzy rules.

The shape of the membership function of the fuzzy output calculated with the commonly used reasoning methods (e.g. "max-min-inference" or "max-prod-inference") is generally different from the shape of the membership functions of the premises and conclusions. If, for instance, all membership functions of the conclusions are fuzzy numbers, the membership function of the fuzzy output is generally not a fuzzy number.

An inference method is expected to evaluate a set of fuzzy rules corresponding to the human way of approximate reasoning. Due to the fact that human beings are able to process only such fuzzy sets that might be properly adjoined to linguistic values, only this kind of fuzzy sets are appropriate inputs of fuzzy systems. Since the membership functions of the premises and conclusions are user-defined to mark linguistic values e.g. as fuzzy numbers, they might be viewed as understandable fuzzy sets. Considering in particular dynamic fuzzy systems that feedback the fuzzy output, it has to be guaranteed that the inference maps understandable fuzzy inputs onto an understandable fuzzy output.

In the following, a new fuzzy inference method called "inference with interpolating rules" is presented that guarantees an output of a fuzzy system belonging to the same class of fuzzy sets as the fuzzy inputs. In this contribution, triangular fuzzy numbers are chosen as understandable fuzzy sets. In the last chapter, it is shown that dynamic fuzzy systems feedbacking the fuzzy output produce suitable results with this new inference method. Furthermore, a new stability definition for dynamic fuzzy systems and an approach for stability analysis are presented.

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2 Inference with interpolating rules

The rules of the fuzzy rule set are considered as "fundamental rules". Together with the fuzzy inputs the fundamental rules allow to determine interpolating rules. An interpolation between premises and conclusions of the fuzzy rule set yields the interpolating premise and the interpolating conclusion of the interpolating rule. Finally, the fuzzy output results from the interpolating rule considering the fuzzy input.

2.1 Assumptions

The membership functions of all premises, conclusions, and inputs have to belong to a class of functions that can be described with a finite number of parameters. In this contribution triangular fuzzy numbers $\langle v_0; v_1, v_2 \rangle$ are used that are clearly defined with their center v_0 and the coordinates v_1 and v_2 of their left and right foot, respectively. An interpolation is only possible if the coordinates of the left and right foot as well as the center of the adjacent premise membership functions are different. Furthermore, it is presupposed that the centers con_0^i of all conclusions Con^i with the coordinates $\langle con_0^i; con_1^i, con_2^i \rangle$ are different from each other and that $con_1^i \le con_1^j$ as well as $con_2^i \le con_2^j$ follows from $con_0^i < con_0^j$ with $\mathrm{i} \neq \mathrm{j}$. These last two conditions represent no restriction to the method and are only introduced to avoid the consideration of some special cases affecting the transparency of this contribution.

2.2 Determination and evaluation of an interpolating rule

The following two fuzzy rules with one input and one output are used to explain the new inference mechanism, the extension to multiple input multiple output systems is straightforward:

IF "Input is small" THEN "Output is large" (1)
IF "Input is large" THEN "Output is small"

The membership functions of the linguistic values "small" and "large" of the input are triangular fuzzy numbers $A < is_0; is_1, is_2 >$ and $B < il_0; il_1, il_2 >$, the linguistic values "small" and "large" of the output are the triangular fuzzy numbers $C < os_0; os_1, os_2 >$ and $D < ol_0; ol_1, ol_2 >$, respectively. Considering a fuzzy input $Inp < inp_0; inp_1, inp_2 >$, the parameters of the interpolating premise $IP < p_0; p_1, p_2 >$ and the interpolating conclusion $IC < c_0; c_1, c_2 >$ of the rule set (1) have to be determined. A measure for the distance between two fuzzy numbers

$$d(A_1, A_2) = center(A_1) - center(A_2)$$

and related distances of *Inp* to *A* and *B*

$$\tilde{a} = \frac{d(Inp, A)}{d(B, A)} = \frac{inp_0 - is_0}{il_0 - is_0}, \qquad \tilde{b} = \frac{d(Inp, B)}{d(A, B)} = \frac{inp_0 - il_0}{is_0 - il_0} = 1 - \tilde{a}.$$

are introduced. The proportion of the two rules IF "Input is small" THEN "Output is large" and IF "Input is large" THEN "Output is small" to the interpolating rule IF "Input is *IP*" THEN "Output is *IC*" correspond to the related distances $\tilde{a}$ and $\tilde{b} = 1 - \tilde{a}$. Thus, the parameters of *IP* and *IC* are

$$p_0 = \tilde{a} \cdot il_0 + (1 - \tilde{a}) \cdot is_0,\ \ p_1 = \tilde{a} \cdot il_1 + (1 - \tilde{a}) \cdot is_1,\ \ p_2 = \tilde{a} \cdot il_2 + (1 - \tilde{a}) \cdot is_2;$$
$$c_0 = \tilde{a} \cdot os_0 + (1 - \tilde{a}) \cdot ol_0,\ \ c_1 = \tilde{a} \cdot os_1 + (1 - \tilde{a}) \cdot ol_1,\ \ c_2 = \tilde{a} \cdot os_2 + (1 - \tilde{a}) \cdot ol_2.$$

The center of the input inp_0 is always equivalent to the center of the interpolating premise *IP*. If *INP* is included in *IP*, *INP* is a fuzzy set with the same center and at least the same specificity (defined in [4]) as *IP*. Therefore, it is straightforward to choose the output $Out < out_0; out_1, out_2 >$ equivalent to *IC* (fig. 1).

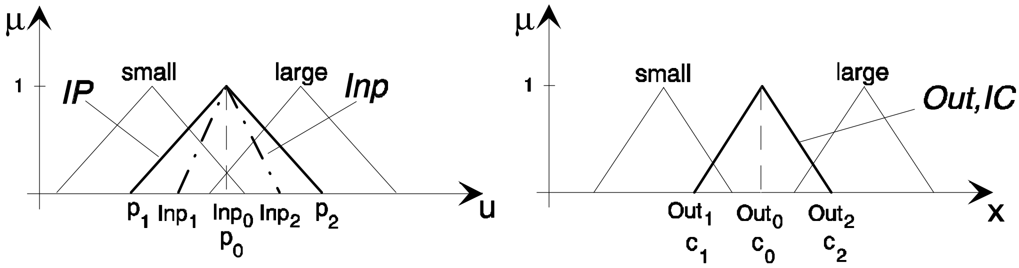

Figure 1. Input is included in the interpolating premise

If the fuzzy input *Inp* is not included in the interpolating premise (see e.g. fig. 2), one or both feet are outside of the support of *IP*. Hence, the parameter of an outlying output foot is appropriately calculated by interpolation between the feet of the membership functions of the linguistic values "small" and "large" of the output as follows:

$$out_1 = \tilde{a}_1 \cdot os_1 + (1 - \tilde{a}_1) \cdot ol_1 , \qquad out_2 = \tilde{a}_2 \cdot os_2 + (1 - \tilde{a}_2) \cdot ol_2 ,$$

$$\tilde{a}_1 = \begin{cases} inp_2 > p_2 : & \dfrac{inp_2 - is_2}{il_2 - is_2} \\ inp_2 \le p_2 : & \tilde{a} \end{cases} \qquad \tilde{a}_2 = \begin{cases} inp_1 < p_1 : & \dfrac{inp_1 - is_1}{il_1 - is_1} \\ inp_1 \ge p_1 : & \tilde{a} \end{cases} .$$

Figure 2 depicts the output *Out* in case of an input *Inp* with only its right foot inp_2 outside of the support of *IP*. Consequently, the left foot out_1 of the output membership function is outside the support of *IC*, whereas the right foot out_2 of the output is equivalent to the right foot c_2 of *IC*. Obviously, moving the right foot of *Inp* causes a relocation of the left foot of *Out* due to the cross-over of the rules (1).

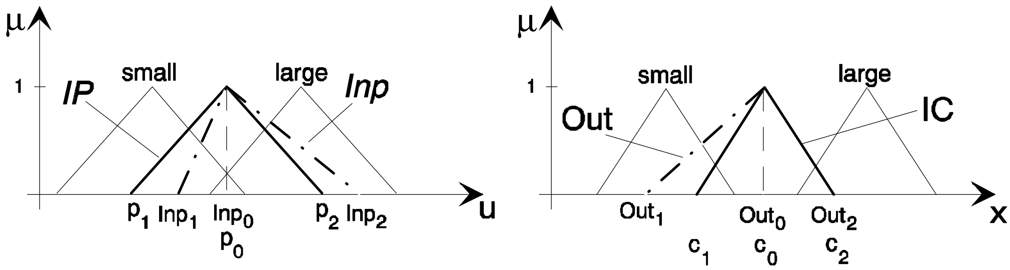

Figure 2. Input is not included in the interpolating premise

Two adjacent membership functions of the input define an interpolation domain. In order to avoid that the transition of the input from one interpolation domain to another is not continuous, fuzzy interpolation domains have to be established by allocating continuous membership functions to each interpolation domain. To sum up this section, the presented inference method guarantees a continuous mapping of understandable fuzzy inputs onto an understandable fuzzy output.

3 Stability Analysis of Fuzzy Systems

In this section, dynamic fuzzy systems feedbacking the fuzzy output (fig. 3) are considered. The inference method described above is used to map the fuzzy input $u_k, y_{k-1}, \dots y_{k-n}$ onto the fuzzy output y_k according to a set of rules like

IF u_k is "small" AND y_{k-1} is "large" AND ... y_{k-n} is "medium" THEN y_k is "large".

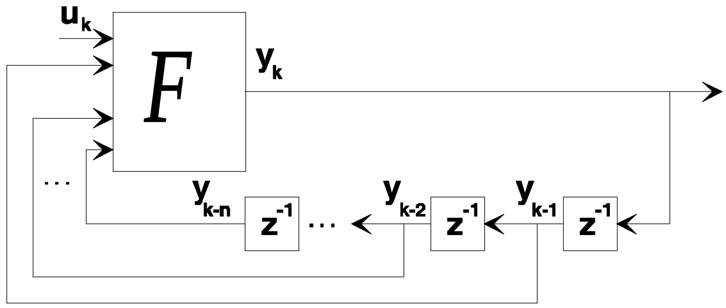

Figure 3. Dynamic Fuzzy System

To show the basic behavior of such a fuzzy system and to come to an appropriate stability definition, it is sufficient to look at a simple undriven fuzzy system only described by the following two rules

IF y_{k-1} is "negative" THEN y_k is "positive"
IF y_{k-1} is "positive" THEN y_k is "negative".

The interpolation domain spanned by the two linguistic values "y_{k-1} is positive" and "y_{k-1} is negative" is the universe of discourse of the linguistic variable y_{k-1}. The membership functions defined on the input domain are shown in figure 4. Depending on the output membership functions, the system shows different dynamic behavior. Given the output membership functions of figure 5a, we obtain system 1 which is stable since the fuzzy output converges to the fuzzy number $y_\infty = \langle 0; -2, 2 \rangle$ for any initial fuzzy state. Figure 5b depicts the fuzzy output resulting from a crisp initial state $y_0 = \langle 2; 2, 2 \rangle$.

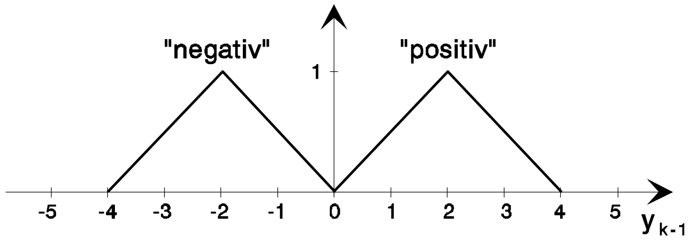

Figure 4. Membership functions of the input domain

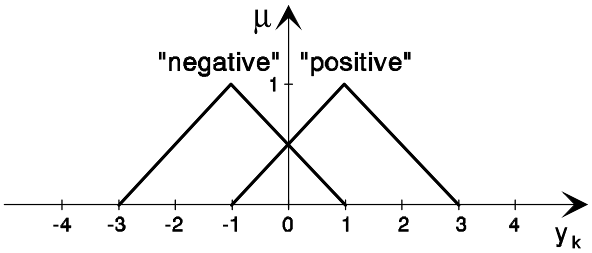

Figure 5a. Output domain membership functions

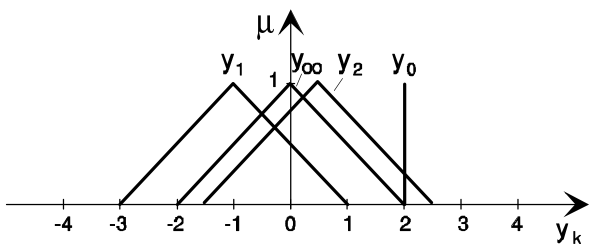

Figure 5b. Simulation of the fuzzy system 1

The output membership functions of system 2 shown in figure 6 cause an unstable system behavior. Although the center of the output converges to 0 for any initial state, its left and right foot move to infinity (fig. 7). Since the output is getting fuzzier with every step, the specificity of the output vanishes for $k \to \infty$.

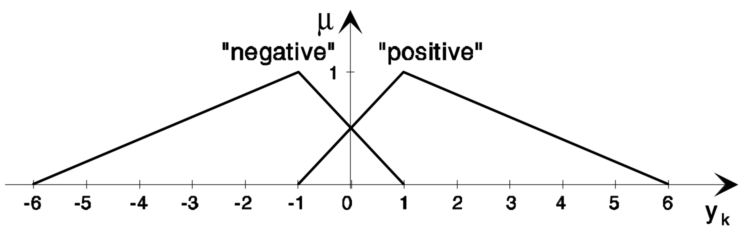

Figure 6. Too fuzzy output membership functions cause an unstable system behavior for system 2

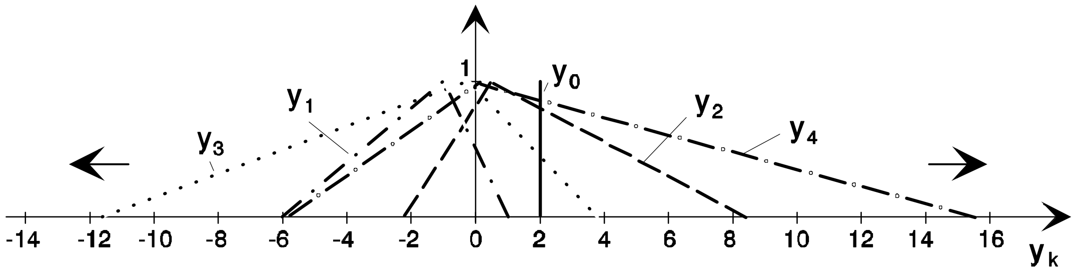

Figure 7. Specificity of the fuzzy output decreases with each step

These simple examples suggest the following stability definition for dynamic fuzzy systems:

An equilibrium point of a fuzzy system marked by a crisp value R_0 is (asymptotically) stable iff

- R_0 is an (asymptotically) stable equilibrium point for the center of the output y_k
- the feet of the fuzzy output stay in a bounded environment of R_0.

The equilibrium point of the two examples above is marked by R_0=0. System 1 has one asymptotically stable equilibrium point, whereas the equilibrium point of system 2 is unstable.

Since it is sufficient to examine the mapping of the crisp parameters of the fuzzy input onto the crisp parameters of the fuzzy output, usual methods for the stability analysis of nonlinear systems can be applied. If the membership functions defined on the input domain of the linguistic values $y_{k-1},\dots y_{k-n}$ are fuzzier than the membership functions defined on the output domain of the linguistic value y_k, it is only necessary to analyze the mapping of the centers of the fuzzy input onto the fuzzy output. With a constant fuzzy u_k it results a discrete nonlinear system described by

$$\mathrm{y}_{\mathrm{k}}^{0} = f\left(\mathrm{y}_{\mathrm{k-1}}^{0},\dots \mathrm{y}_{\mathrm{k-n}}^{0}\right)$$

with the centers $\mathrm{y}_{\mathrm{k}}^{0},\mathrm{y}_{\mathrm{k-1}}^{0},\dots \mathrm{y}_{\mathrm{k-n}}^{0}$ of the fuzzy output y_k and its delays $y_{k-1},\dots y_{k-n}$. To analyze such a system, methods based on common stability analysis approaches may be used. The "Convex Decomposition" [2,3] as an efficient numerical stability analysis method has been successfully applied to dynamic fuzzy systems.

Further investigations will concentrate on analysis (e.g. controllability) and the synthesis of fuzzy logic controllers with dynamic fuzzy systems as qualitative process models.

References

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